NATURAL NUMBERS or COUNTING NUMBERS   [1, 2, 3, 4, 5 …]
- Positive numbers beginning with 1 and increasing by 1 or multiples of 1.

WHOLE NUMBERS   [0, 1, 2, 3, 4, 5 …]
- Natural numbers with the addition of zero ("0").

INTEGERS   [… -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5 …]
- Positive and negative natural numbers and zero.

RATIONAL NUMBERS or FRACTIONS   Examples: 1, 1.2, $5/3$, etc…
- All numbers that can be written as the division of two integers.
- For instance: $3/4$ may be written $3 \div 4$. The top number of a fraction is the numerator and the bottom number of a fraction is the denominator. The decimal equivalent of this fraction is 0.75. In surveying we often convert fractions to their decimal equivalents. This facilitates the use of the calculator for arithmetic calculations. It should be noted that the number of significant figures retained in the calculator is important to the final answer obtained.
- Note: the decimal equivalents of rational numbers are either terminating or repeating decimals.)

IRRATIONAL NUMBERS   Examples: $\sqrt{2}$, π, etc…
- Numbers that ***cannot*** be written as the division of two integers.
- Note: the decimal equivalents of irrational numbers are non-terminating and non-repeating decimals.)

RECIPROCALS   Example: $4/3$ and $3/4$ are reciprocals.
- The reciprocal of any fraction is a fraction where the numerator and denominator are reversed.
- The product of the original fraction and its reciprocal equals 1.
- Note: any number can be written as a fraction by putting it over 1.
    Examples: $1 = 1/1$ and $5.2 = 5.2/1$

REAL NUMBERS
- All rational and irrational numbers.

IMAGINARY NUMBERS   Example: $\sqrt{-4}$ or $2i$ where $i = \sqrt{-1}$
- Numbers that can be expressed in written form but which have no "usable" mathematical value.

COMPLEX NUMBERS   Example: $3 + 4i$ or $(3, 4i)$ where $i = \sqrt{-1}$
- All real and imaginary numbers.

GREATEST COMMON FACTOR
- The largest number that divides (without a remainder) every number in a given set of natural numbers.
    - Example: what is the greatest common factor of 18 and 24?
        - the factors of 18 are 1, 2, 3, 6, 9 and 18
        - the factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24
        - therefore, the greatest common factor of 18 and 24 is 6.

A few quick definitions for review…

- When one adds,   the answer is the <u>sum</u>,
- When one subtracts,   the answer is the <u>difference</u>,
- When one multiplies,   the answer is the <u>product</u>,
- When one divides,   the answer is the <u>quotient</u>,

A review of some "properties" of number calculations…

COMMUTATIVE PROPERTY
- For addition and multiplication the order of operation is immaterial.

$$A + B = B + A \quad \text{and} \quad A * B = B * A$$

ASSOCIATIVE PROPERTY
- For addition and multiplication of three or more numbers, parentheses may be placed around any two adjacent numbers without changing the result.

$$(A + B) + C = A + (B + C) \quad \text{and} \quad (A * B) * C = A * (B * C)$$

- Note: only addition or multiplication is being done in each equation – when mixing arithmetic functions, other rules must be used…

DISTRIBUTIVE PROPERTY
- For addition and multiplication…

$$A * (B + C) = A * B + A * C$$

- For now, just be aware that the distributive property exists. We will be examining this when we study "factoring for simplifying and rearranging equations".

Addition and subtraction, and multiplication and division, are related operations. Any subtraction problem can be written as an addition problem by changing the sign of the number being subtracted and then adding. Any division problem can be written as a multiplication problem by taking the reciprocal of the divisor (denominator) and multiplying it by the dividend (numerator). Therefore, the properties already described can be applied to their related operation once the subtraction or division is rewritten as addition or multiplication respectively.

$$A - B = A + (- B) \quad \text{and} \quad A \div B = A * (1 / B)$$

A review of operations with "signed" numbers…

- If either the numerator (dividend) or the denominator (divisor), ***but not both***, of a fraction is negative, then the entire fraction may be written as a negative.

- To add two numbers when their signs are the same (positive or negative) add their absolute values (the value of the numbers disregarding their signs).
  The sum has the same sign as that of each of the numbers being added.
- To add two numbers with different signs (one positive and the other negative) subtract the smaller absolute value from the larger absolute value.
  The sum has the same sign as the number with the larger absolute value.

- The product of two numbers with like signs is always positive.
- The product of two numbers with opposite signs is always negative.

- Division is the same as multiplication by the reciprocal of the divisor, therefore, the rules of multiplication of signed numbers also apply to the division of signed numbers.

ORDER OF OPERATIONS in algebraic expressions…

- ***P*** Parentheses – evaluate the information within parentheses or brackets.
- ***E*** Exponents – evaluate all exponents and roots.
- ***M*** Multiplication and
- ***D*** Division – evaluate all multiplication and divisions from left to right.
- ***A*** Addition and
- ***S*** Subtraction – evaluate all additions and subtractions from left to right.

- Note: If the expression contains nested parentheses – one pair of parentheses within another pair – evaluate the information in the innermost parentheses first.

RATIO

- The quotient of two numbers, usually written as a fraction where the denominator is never zero.

  Example 1: write the ratio of 8 is to 12 as a fraction… $8/12$
  Example 2: write the ratio of 2 is to 3 as a fraction… $2/3$

- Ratios are equal if they are the same when expressed as fractions in reduced form. [The ratios in examples 1 and 2 are equal, since $8/12 = 2/3$]

- When a ratio expresses the amount of one thing to the units of another, we refer to it as a <u>rate</u>.
  Example: an automobile traveling at a rate of 55 miles per hour expresses the ratio of miles (distance) to one hour (a unit of time).

PROPORTION

- A statement that two ratios are equal.

  Examples of proportions would be…
  $$\frac{4}{6} = \frac{2}{3}, \qquad \frac{3}{4} = \frac{75}{100}, \qquad \frac{5}{10} = \frac{10}{20}$$

PERCENT

- Percent means <u>per hundred</u> and indicates a ratio of a number to 100.

  Example 1: 75% means… $75/100$
  Example 2: 150% means… $150/100$

The meaning of percent therefore allows us to change any percent to a common fraction or a decimal.

- Rule: To change a percent to a decimal number remove the % sign and move the decimal point two places to the left. Conversely, when changing a decimal number to a percent, move the decimal point two places to the right and add the % sign.
- Rule: To change a fraction to a percent, first change the fraction to a decimal number and then change the decimal number to a percent.